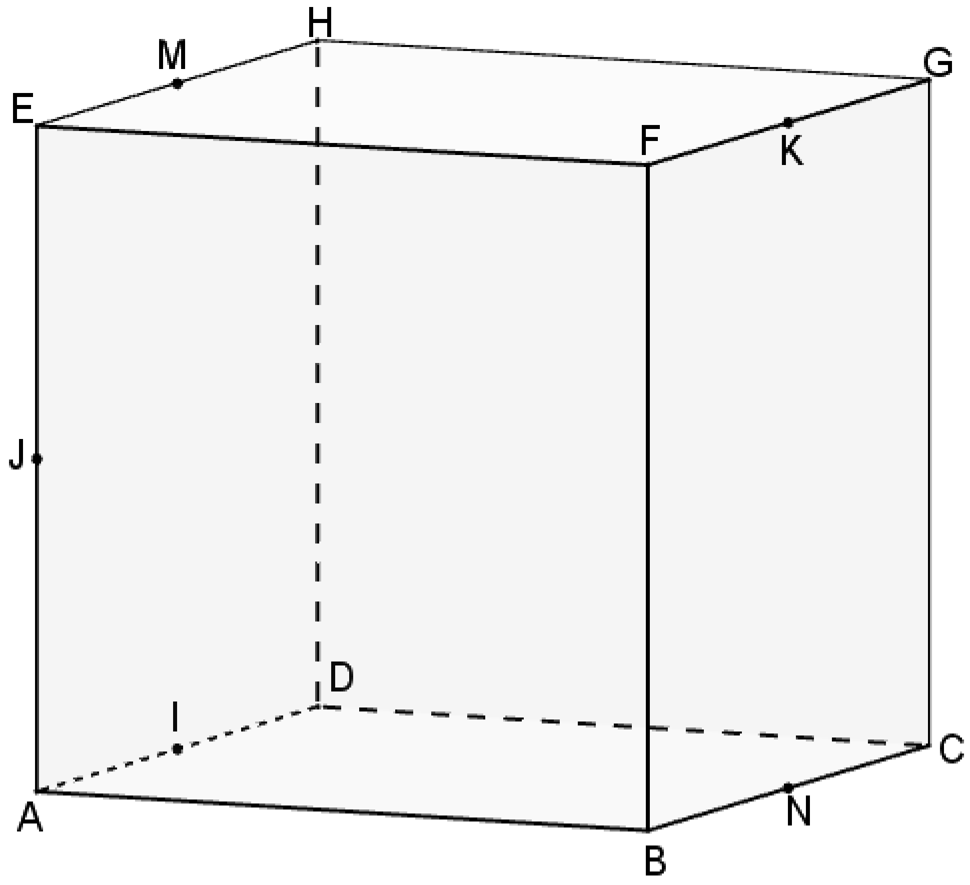
ABCDEFGH is a cube of side a.

I, J, K, L, M and N are the respective midpoints of [AD], [AE], [FG], [EH] and [BC].

Prove each given affirmation.



**1. (FB) and (EC) are not coplanar.**

Two lines are coplanar if they belongs to the same plane i.e. they are intersecting or parallel.

Solution:

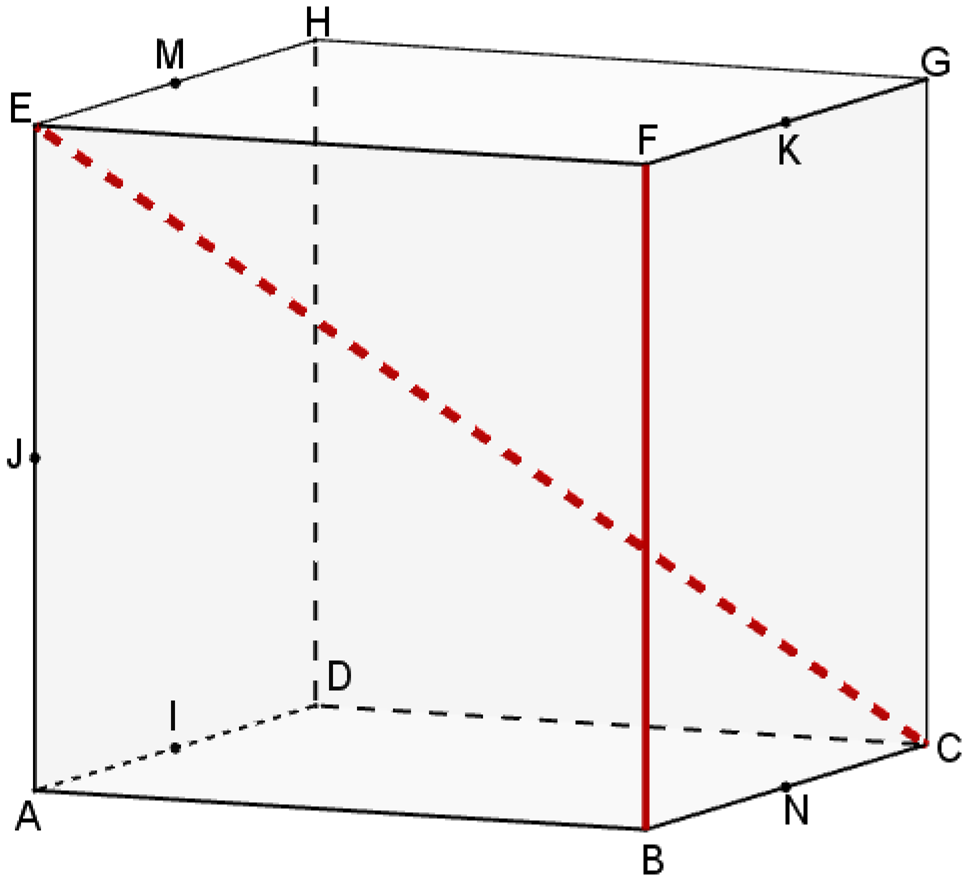
- (FB)//(GC)
- (EC) cuts (GC)

So (FB) and (EC) are not parallel.

- (FB)⊂(EFBA)
- (EC) cuts (EFBA) at E
- E ∉(FB)

So (FB) and (EC) are not intersecting lines

So (FB) and (EC) are not coplanar.



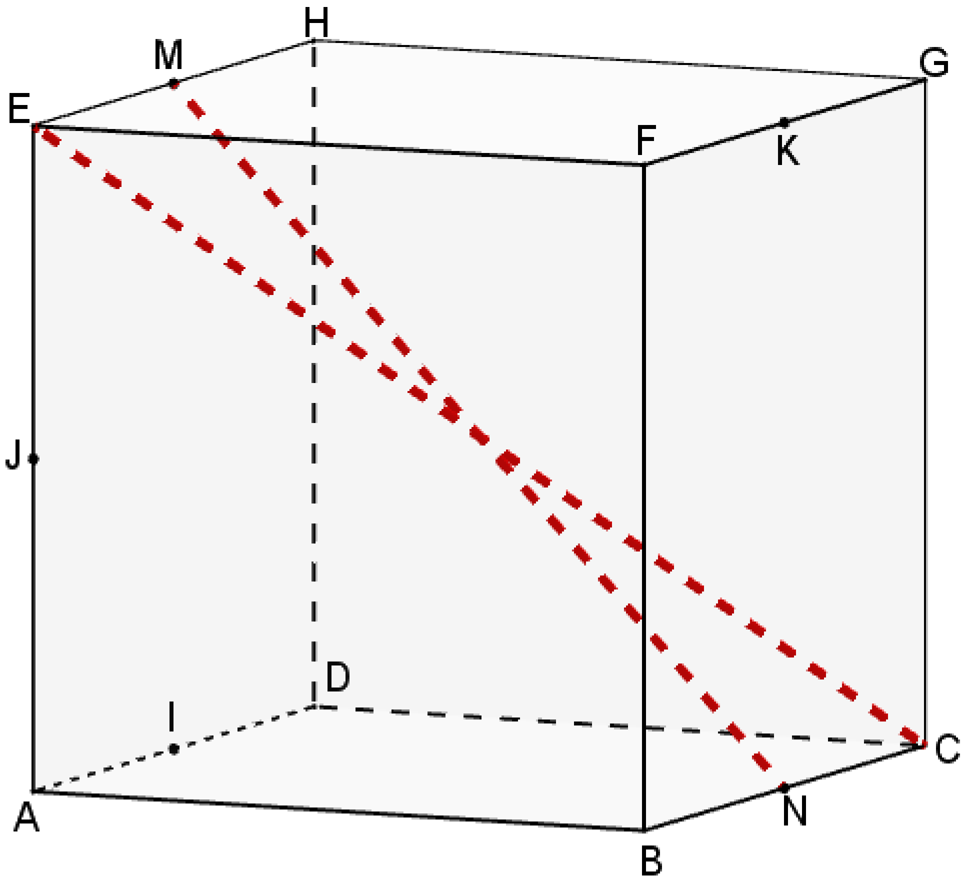

**Self task:**
**Show that (HG) and (FD) are not coplanar.**

Two lines are coplanar if they belongs to the same plane i.e. they are intersecting or parallel.

**2. (MN) and (EC) are intersecting lines.**

Solution:

- (EC)⊂(EHCB)
- M∈(EH) so M∈(EHCB)
  N∈(BC) so N∈(EHCB)
  then (MN)⊂(EHCB)
- Then (MN) and (EC) are coplanar
- Hence (MN) cuts (EC)




**Self task:**
**Show that (MB) cuts (HC).**

➢ To prove that a line is perpendicular to a plane, it is sufficient to prove that it is orthogonal to two intersecting lines of this plane.
➢ If a line is perpendicular to a plane, then it is orthogonal to all the lines of this plane.

**3. (EF) and (GB) are orthogonal.**

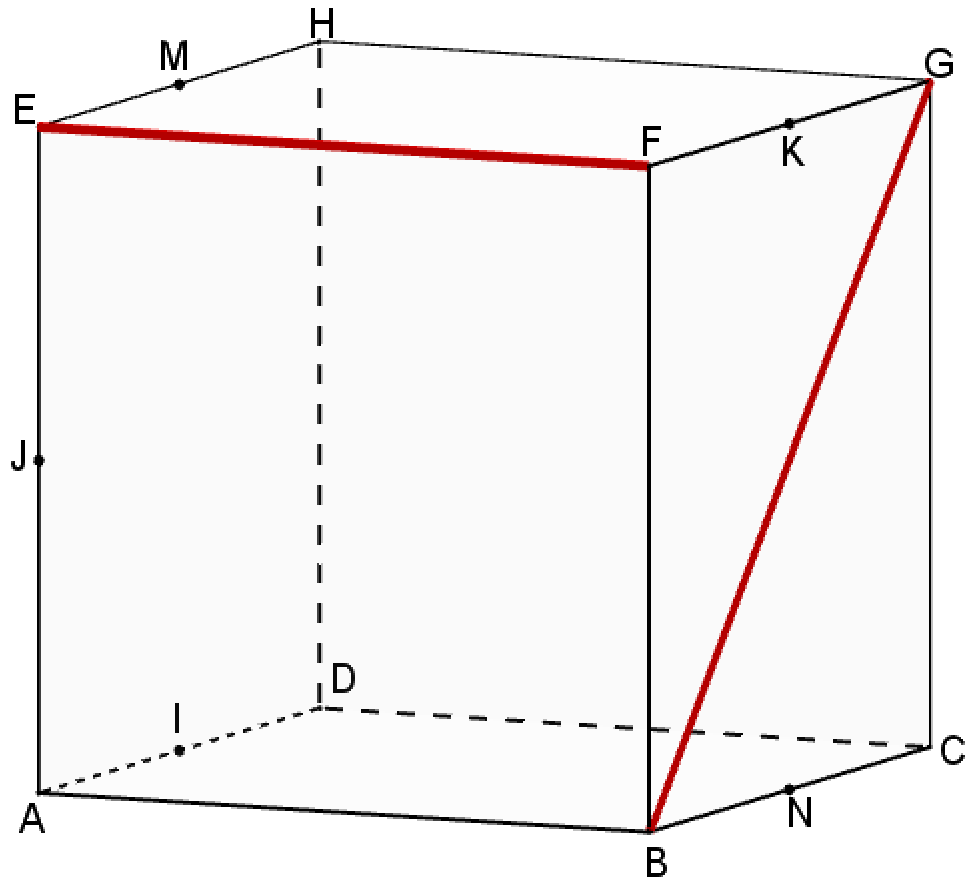
Solution:

- (EF)⊥(FG) since (EFGH) is a square.
- (EF)⊥(FB) since (EFBA) is a square.

So (EF)⊥(FGCB)

- (EF)⊥(FGCB)
- (BG)⊂(FGCB)

So (EF) is orthogonal to (BG)




**Self task:**
**Show that (HE) is orthogonal to (AF).**

**4. (EGCA) and (ABCD) are perpendicular.**

Solution:

- (EA)⊥(AB) since (EFBA) is a square.
  (EA)⊥(AD) since (EHDA) is a square.
  So (EA)⊥(ABCD)

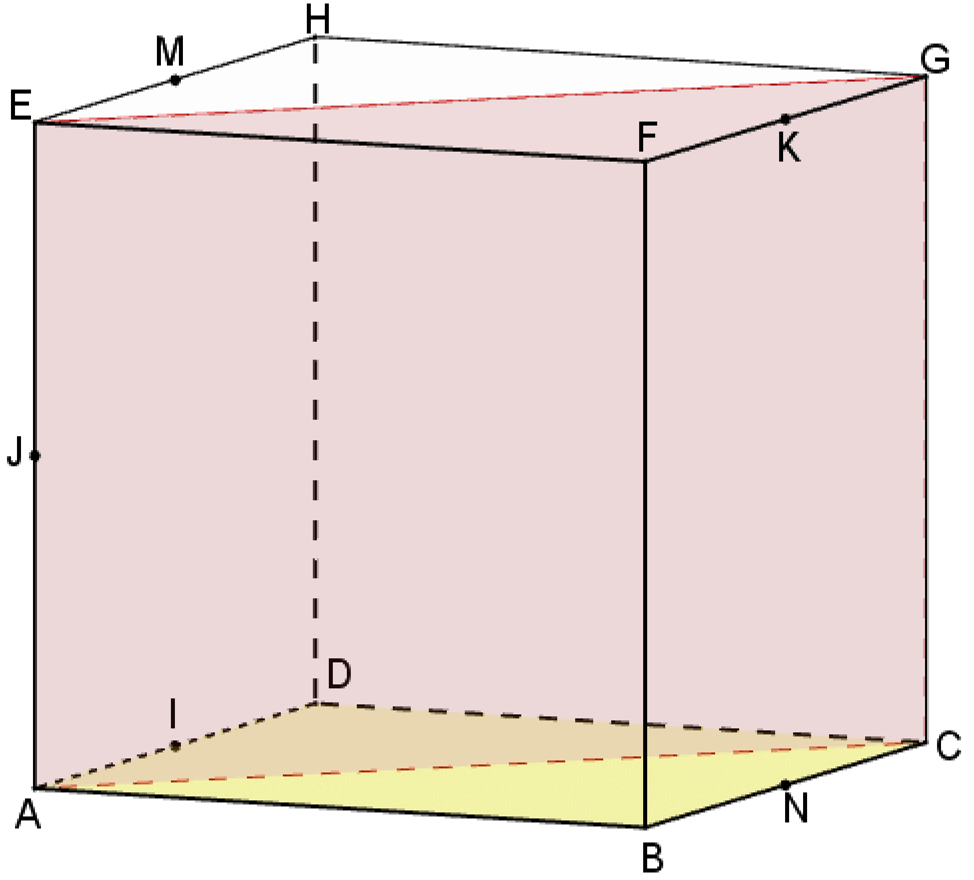
- (EA) ⊥(ABCD)
  (EA)⊂(EGCA)
  so, (EGCA) is perpendicular to (ABCD).

**Self task:**
**Show that (HFBD) is orthogonal to (EHGF).**

- ➢ To prove that a plane (P) is perpendicular to another plane (Q):
- ✓ If (P)//(R) and (Q)⊥(R) then (P)⊥(Q)

Or

- ✓ If (P) contains a line that is perpendicular to (Q), then (P) ⊥(Q).



**5. The angle between the line (BM) and the plane (AEHD) is ≅41.81°**

Solution:

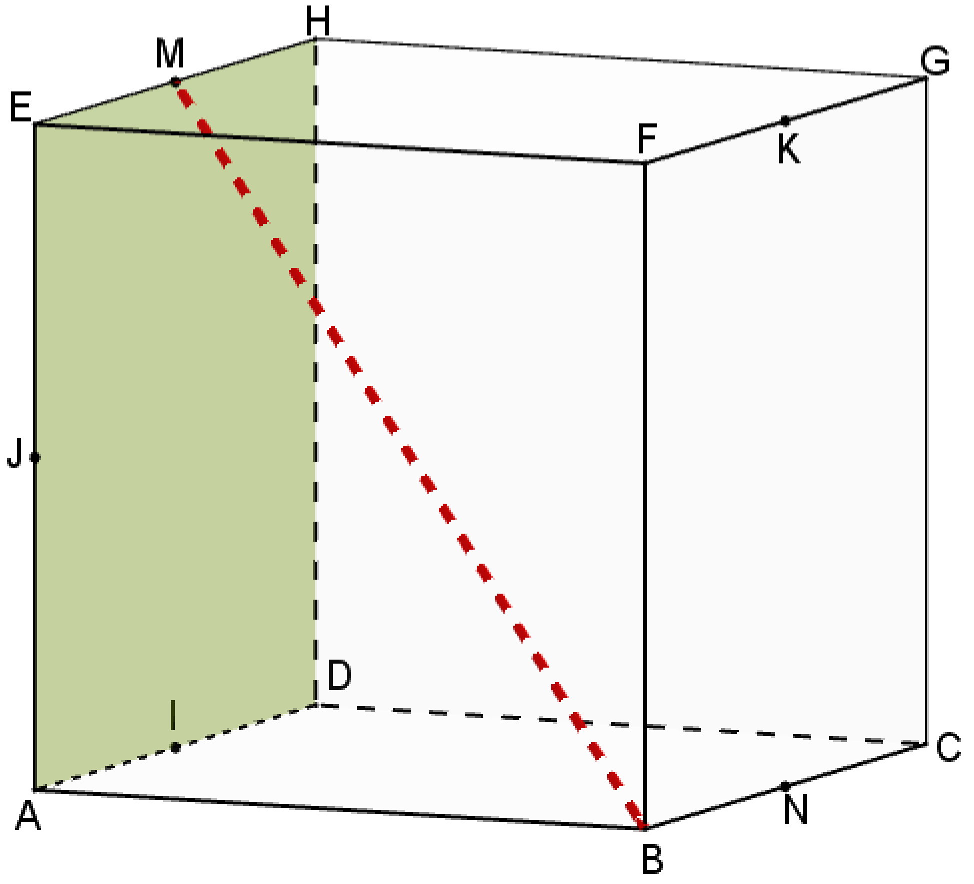
- (BM)∩(AEHD)={M}
- B∈(BM)
- (BA)⊥(AD) since ABCD is a square
  (BA)⊥(AE) since EFBA is a square
  then, (BA)⊥(AEHD) at A
- ∠((BM);(AEHD))=∠AMB
- In the right triangle AMB at A:

Tan(AMB)=$\frac{AB}{AM}=\frac{a}{\frac{\sqrt{5}a}{2}}=\frac{2}{\sqrt{5}}$

∠AMB=$\tan^{-1}\frac{2}{\sqrt{5}}=41.81°$

**Self task:**
**Find the angle between (EB) and the (ABCD).**

- To find the angle between a line (d) and a plane (P):
  - Find the intersection point M of (d) and (P).
  - Find a point A of (d).
  - Find the orthogonal projection H of A on (P).
  - Tha angle between (d) and (P) is the angle AMH



**6. The dihedral angle between (MBC) and (ABCD) is 45°.**

Solution:

- (BC)=(MBC)∩(ABCD)
- (MBC)=(MEBC) so (EB)⊂(MBC)

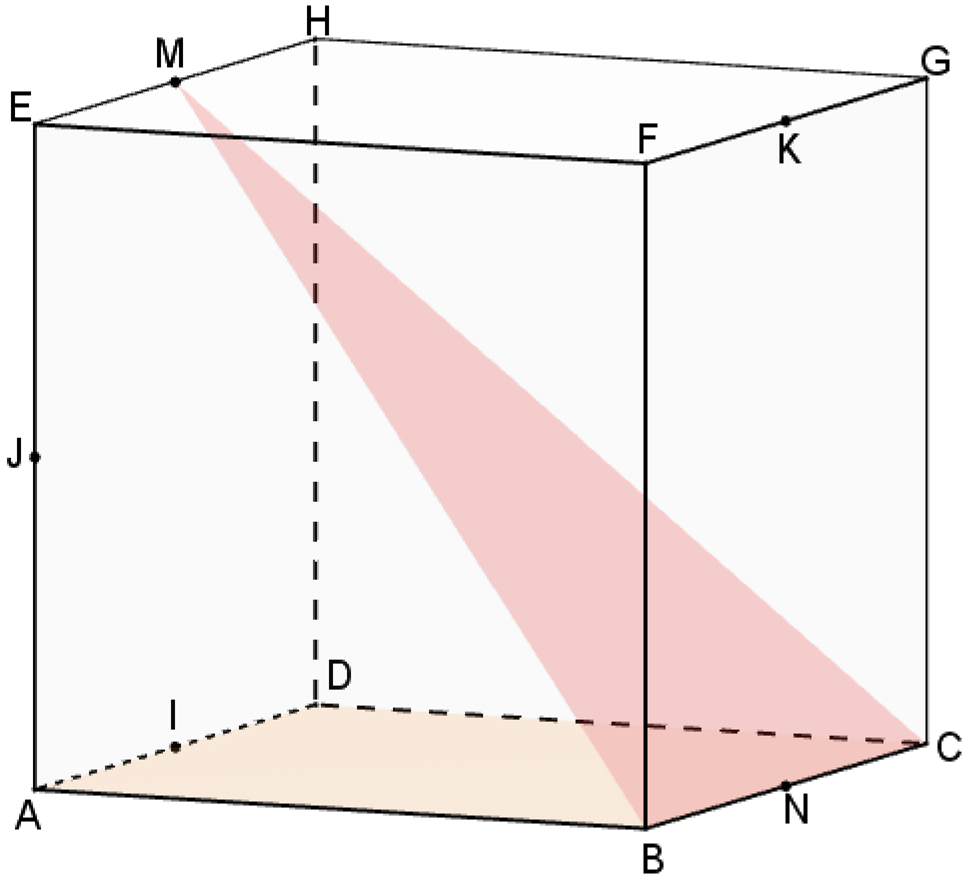
(EB)⊥(BC) since:
(BC) ⊥(AB)
(BC) ⊥(FB) so (BC) ⊥(AEFB)
But (EB)⊂(AEFB)
Hence (EB) ⊥(BC)

- (AB)⊂(ABCD) and (AB) ⊥(BC)
- The dihedral angle between (MBC) and (ABCD) is ∠EBA

But AEFB is a square, then ∠EBA=45°

**Self task:**
**Find the angle between (HIG) and the (HGCI).**

- To find the angle between a plane (P) and a plane (Q):
  - Find the intersection line (d) of the two planes.
  - Find (d1) of (P) that is perpendicular to (d).
  - Find (d2) of (Q) that is perpendicular to (d).
  - The angle between (P) and (Q) is equal to that between (d1) and (d2).



➢ To prove that (P) is the mediator plane of a segment:
Find 3 points of (P) that are equidistant from the extremities of the segment.

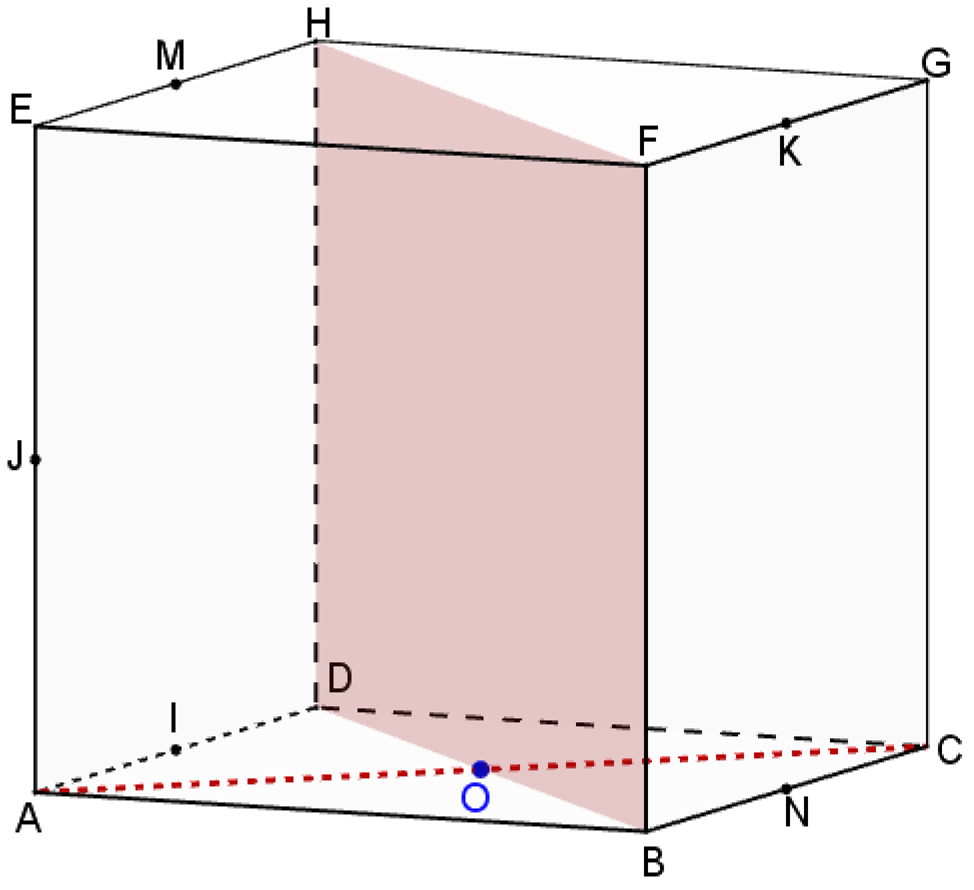
**7. (HFBD) is the mediator plane of [AC]**

Solution:

- B∈(HFBD) and BC=BA (ABCD is a square)
- D∈(HFBD) and DA=DC (ABCD is a square)
- F∈(HFBD)

FA=a√2 (diagonal in the square AFBA)
FC=a√2 (diagonal in the square FGCB)
So FA=FC

- Then (HFBD) is the mediator plane of [AC].




**Self task:**
**Show that (EGCA) is the mediator plane of [BD].**